

Effect of loading conditions on the stress concentration factor for a plate

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Abstract: Stress concentration is defined as the localization of high stresses due to the irregularities present in the component and abrupt changes of the cross section. The stress concentration factor is defined as the ratio of maximum stress to nominal stress. To study the effect of loading conditions on the Stress Concentration Factor for a plate, a specimen is selected as a plate having a centrally located circular opening keeping the area of cross section same. Eight different cases are considered for analysis, four of uni-axial and four of bi-axial. From the analysis the results are obtained and are plotted as SCF (Kt) Vs d/w ratio. The Stress concentration factor, Stress induced and the Deflections are not affected by the Tensile or Compressive loading when the loading directions and the boundary conditions are remains same for the same test specimen. The Analytical values of Stress Concentration Factor and that obtained by FEA decreases exponentially with increase in d/w ratio but varies from each other by small negotiable amount in all conditions and the behavior of the graph remains same in all cases except in mixed loading which is almost linear. Also another case study on the pattern of holes is studied as square pattern and diamond pattern. From the results obtained it is clear that square pattern is favorable on diamond as there is a neck formation in diamond which leads to failure of the plate.

Keywords: Stress Concentration Factor, Plate with Hole, Loading Conditions

I. Introduction

Stress concentrators cause high stresses in the structure. There are different formulas for nominal stress, usually it is stress in the absence of concentrator. "Stress concentration is defined as the localization of high stresses due to the irregularities present in the component and abrupt changes of the cross section". In order to consider the effect of stress concentration and find out localized stresses, a factor called stress concentration factor (SCF) is used. It is denoted by Kt and defined as [Bhandari V. B., 2008],

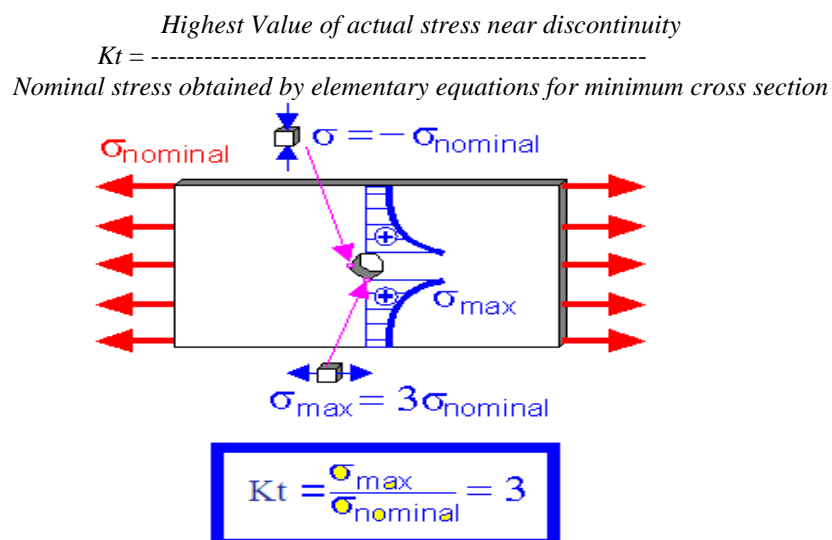


Fig.1 Plate with a Hole

I.1 Geometric Stress Concentration Factor [Sengupta A. K.]:

To account for stress concentration effect, the actual maximum stresses have been determined either experimentally or by using more sophisticated stress analysis methods, such as finite element analysis, for

common types of geometric features. Based on such calculations the geometric stress concentration factors (Kt) are determined for these types of features. "The stress concentration factor is defined as the ratio of maximum stress to nominal stress". The value of the factor Kt varies from 1 to about 3 in most cases. Kt = 1 means no stress concentration, that is, calculated value of stress = actual value of stress. When Kt = 3, the actual stress is three times the calculated value. Fig.1 shows the plate subjected to tensile loading at both sides due to which stresses are induced in a plate. One is $\sigma_{nominal}$ and other is σ_{max} which is 3 times $\sigma_{nominal}$, therefore SCF i.e. Kt is 3.

The formula for the stress concentration factor is [Bhandari V. B., 2008],

$$Kt = \frac{\sigma_{max}}{\sigma_{nominal}}$$

Where,

Kt = Stress Concentration Factor

σ_{max} = maximum stress obtained by analysis in N/mm²

$\sigma_{nominal}$ = nominal stress obtained by the formula, in N/mm²

Nominal stress is given by R. E. Peterson's formula [Bhandari V. B., 2008],

$$\sigma_{nominal} = \frac{P}{A} = \frac{P}{(w - d)t}$$

Where,

P = Tensile load applied in N

w = Width of the plate in mm

d = Width of the opening in mm

t = thickness of the plate in mm.

And the Heywood formula for Analytical stress concentration factor is applicable to Circle only as [Walter D. Pilkey, Deborah F. Pilkey, Peterson's Stress Concentration Factors, 3rd Edition],

$$Kt = 2 + \left(1 - \frac{d}{w}\right)^3$$

Where,

Kt = Stress Concentration Factor

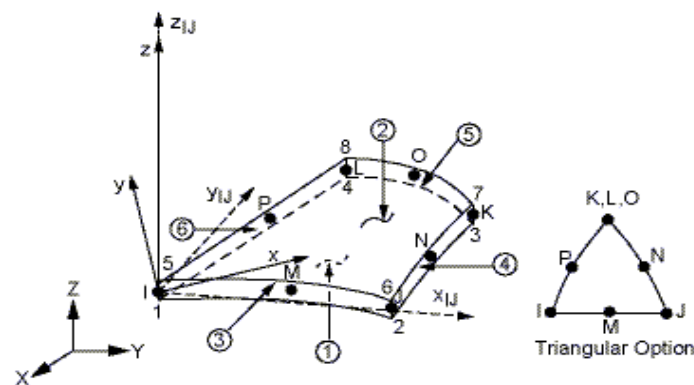
d = Width of the opening in mm

w = Width of the plate in mm

II. Element Selection (8 Node Shell 93) [Ansys Release 11.0 Documentation]:

II.1 DESCRIPTION:

SHELL93 is particularly well suited to model curved shells, the geometry of which is shown in Fig.2. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. The deformation shapes are quadratic in both in-plane directions. The element has plasticity, stress stiffening, large deflection, and large strain capabilities.



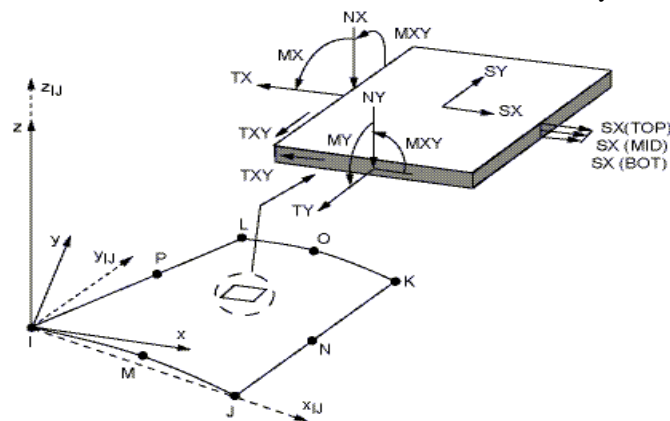
x_{IJ} = Element x-axis if ESYS is not supplied.

x = Element x-axis if ESYS is supplied.

Fig. 2: SHELL93 Geometry

II.2 Assumptions And Restrictions:

- Zero area elements are not allowed. This occurs most often whenever the elements are not numbered properly.
- Zero thickness elements or elements tapering down to a zero thickness at any corner are not allowed.
- The applied transverse thermal gradient is assumed to vary linearly through the thickness.
- Shear deflections are included in this element.
- The out-of-plane (normal) stress for this element varies linearly through the thickness and the transverse shear stresses (SYZ and SXZ) are assumed to be constant through the thickness, these stresses are well shown in Fig.3.
- The transverse shear strains are assumed to be small in a large strain analysis.
- This element may produce inaccurate stresses under thermal loads for doubly curved or warped domains.



x_{IJ} = Element x-axis if ESYS is not supplied.

x = Element x-axis if ESYS is supplied.

Fig. 3: SHELL93 Stress Output

II.3 Product Restrictions:

When used in the product(s) listed below, the stated product-specific restrictions apply to this element in addition to the general assumptions and restrictions given in the previous section.

ANSYS Professional.

- The DAMP material property is not allowed.
- The special features allowed are stress stiffening and large deflection.
- KEYOPT(4) can only be set to 0 (default).

III. Assumptions And Conditions:

To study the effect of loading conditions on the Stress Concentration Factor for a plate with a circular hole, a specimen is selected as a plate having material properties and dimensions as mentioned below.

Material Properties:

- 1) Material Used : Mild Steel
- 2) Material Property : Linear-Elastic-Isotropic, Homogeneous
- 3) Modulus of Elasticity (E): $1.8 \times 10^5 \text{ N/mm}^2$
- 4) Poisson's Ratio (μ): 0.29

Geometric Dimensions [Hwai-Chung Wu, bin Mu, 2003]:

- 1) Length (L): 254 mm
- 2) Width (w): 101.6 mm
- 3) Thickness (t): 10.16 mm

The plate is having a centrally located Circular opening the Diameter of which changes for different Loading Conditions as below,

- 1) Uni-axial
 - a) Tensile
 - i) At one end,
 - ii) At Both ends.
 - b) Compressive
 - i) At one end,
 - ii) At Both ends.
- 2) Bi-axial
 - a) Tensile
 - i) At Both ends.
 - b) Compressive
 - i) At Both ends.
 - c) Mixed
 - i) Width-Tensile, Length-Compressive.
 - ii) Width- Compressive, Length- Tensile.

To study the effect of geometry of opening on SCF for a plate, two boundary conditions are used as two cases in ANSYS. The element used is '8 node Shell 63' and the meshing for analysis is free type having mesh size of 4.

IV. Case Studies Of Loading Conditions:

For different loading conditions as mentioned above the analysis on ANSYS is carried out to find out the maximum stress and deflection. And also the nominal stress and the analytical stress concentration factors are calculated by available formulae's. The results obtained for different loading conditions are tabulated (from Table no.1 to 4) along with graph as under (from Graph no. 1 to 4). One of the loading condition is shown in Fig.4 while the results for it shown in Fig.5.

While the maximum deflections and equivalent stress for other loading conditions at different diameters are shown in Fig.6 and Fig.7.

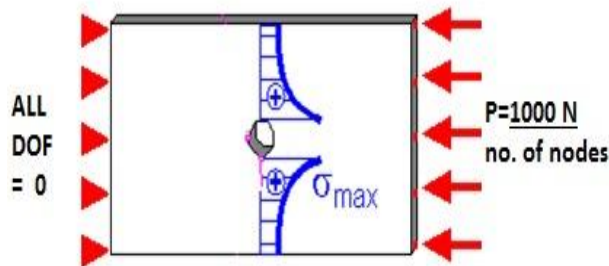


Fig.4: Plate with a Circular Hole Under Uniaxial Compressive Loading at one end only

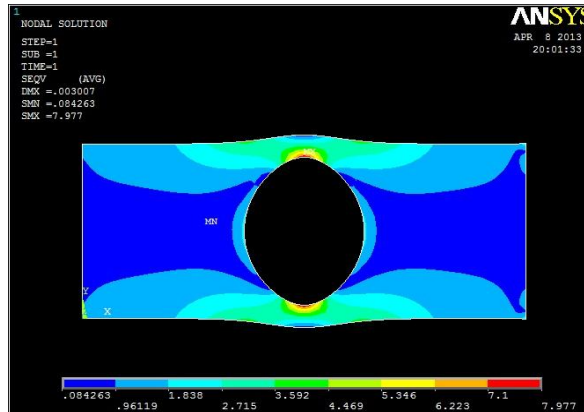
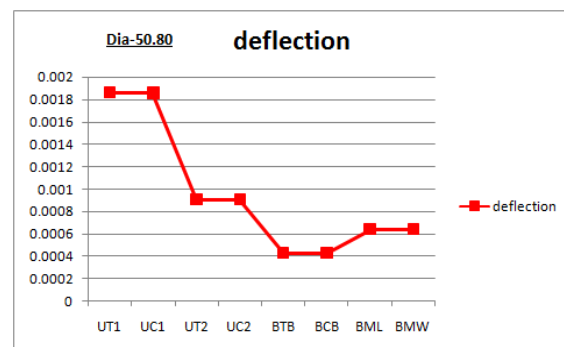
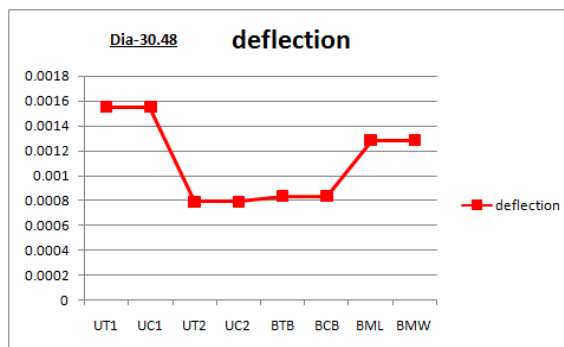
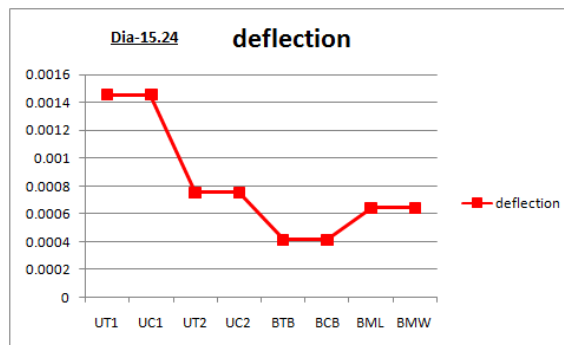


Fig.5: Von Misses Stress for Uniaxial Compressive Loading at one end only



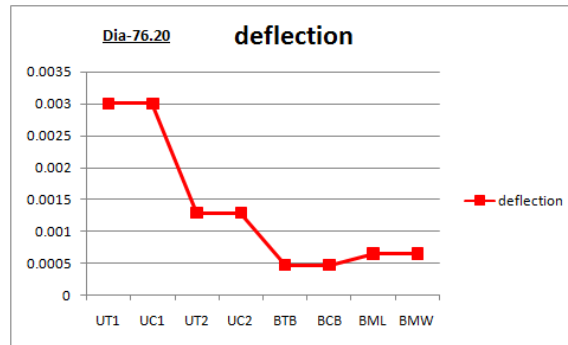
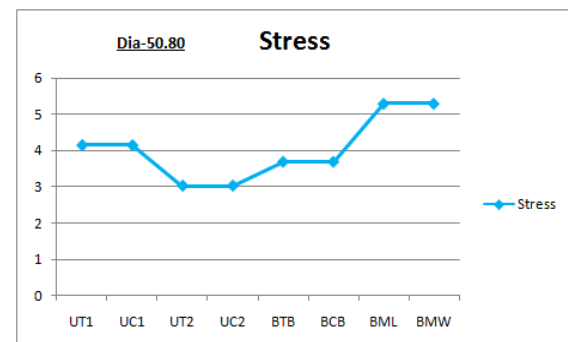
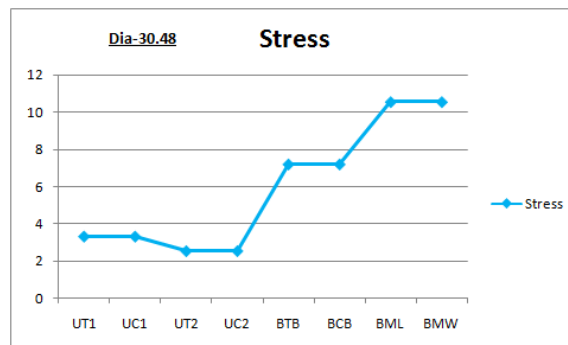
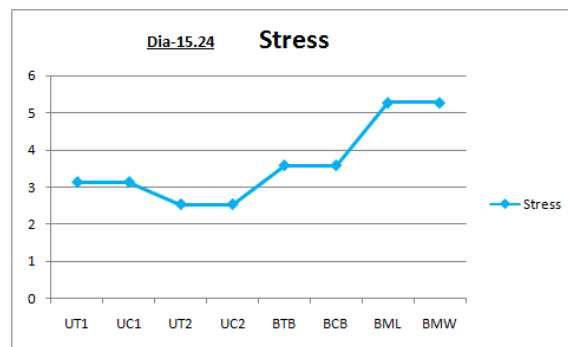


Fig.6: Max. Deflection results for diff. diameters



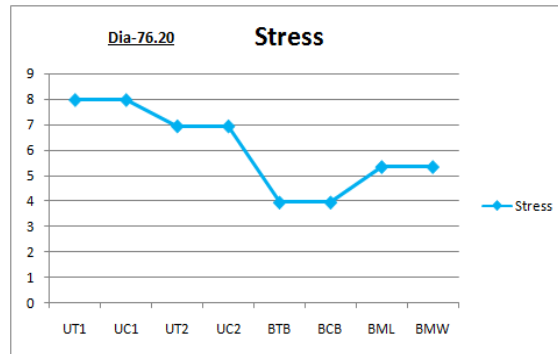


Fig.7: Max. Von Misses Stress results for different diameters

Table 1: Uni-axial Tensile or Compressive loading at one end

Sr. No.	d/w	σ max By FEA	σ nominal	Kt By FEA	Kt Analytical
1	0.15	3.135	1.1397	2.751	2.614
2	0.3	3.321	1.3839	2.399	2.343
3	0.5	4.149	1.9375	2.141	2.125
4	0.75	7.977	3.875	2.058	2.016

Table 2: Uni-axial Tensile or Compressive loading at both ends

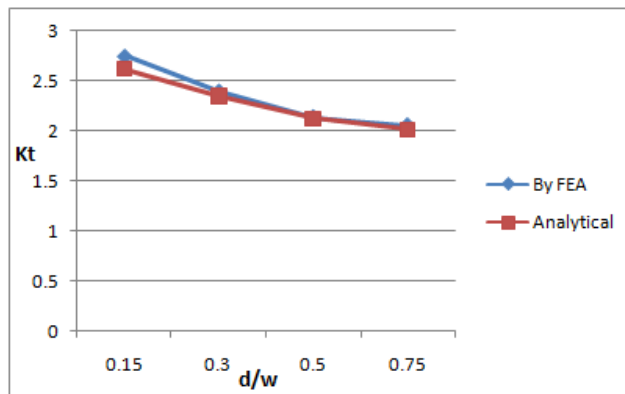
Sr. No.	d/w	σ max By FEA	σ nominal	Kt By FEA	Kt Analytical
1	0.15	2.533	1.1397	2.223	2.614
2	0.3	2.536	1.3839	1.833	2.343
3	0.5	3.023	1.9375	1.561	2.125
4	0.75	6.946	3.875	1.793	2.016

Table 3: Bi-axial Tensile or Compressive loading at both ends

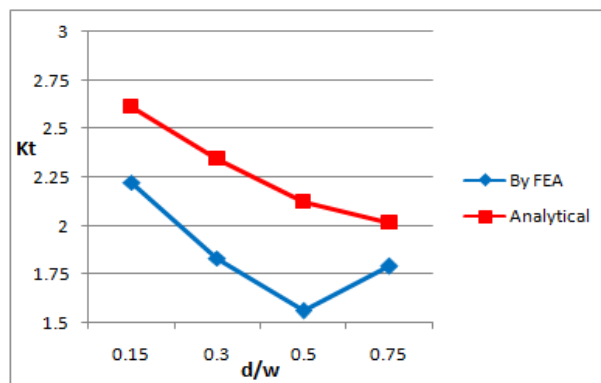
Sr. No.	d/w	σ max By FEA	σ nominal	Kt By FEA	Kt Analytical
1	0.15	3.586	1.1397	3.146	2.614
2	0.3	3.601	1.3839	2.602	2.343
3	0.5	3.687	1.9375	1.903	2.125
4	0.75	3.953	3.875	1.020	2.016

Table 4: Bi-axial Tensile and Compressive Mixed loading

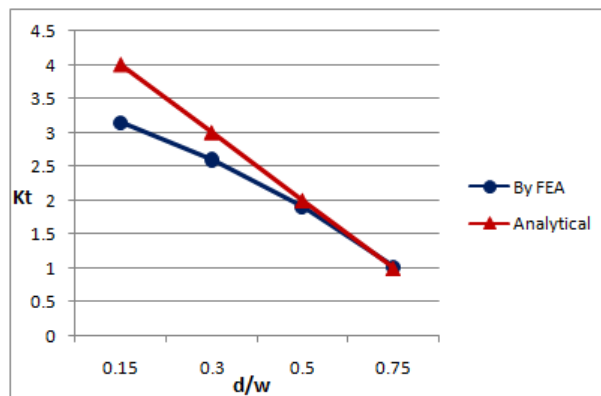
Sr. No.	d/w	σ max By FEA	σ nominal	Kt By FEA	Kt Analytical
1	0.15	5.278	1.1397	4.631	2.614
2	0.3	5.276	1.3839	3.812	2.343
3	0.5	5.303	1.9375	2.737	2.125
4	0.75	5.347	3.875	1.380	2.016



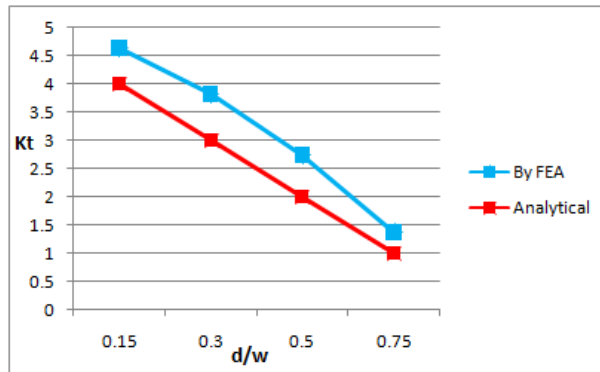
Graph 1: Uni-axial Tensile or Compressive loading at one end



Graph 2: Uni-axial Tensile or Compressive loading at both ends



Graph 3: Bi-axial Tensile or Compressive loading at both ends



Graph 4: Bi-axial Tensile and Compressive Mixed loading

IV.1 Case Study Of Pattern Of Holes:

In this a pattern of four holes of same diameter on a plate is considered at two different orientations one is square pattern and other is of diamond pattern. The tensile load is applied at one end keeping other end of the plate fixed (All DOF= 0). Then observed the results obtained for the displacement vector sum and Von Misses stresses, and draw the conclusion for the same. Fig.8, 9 and 10 shows loading condition, deflection and stress results for square pattern while Fig.11, 12 and 13 shows loading condition, deflection and stress the results for diamond pattern respectively.

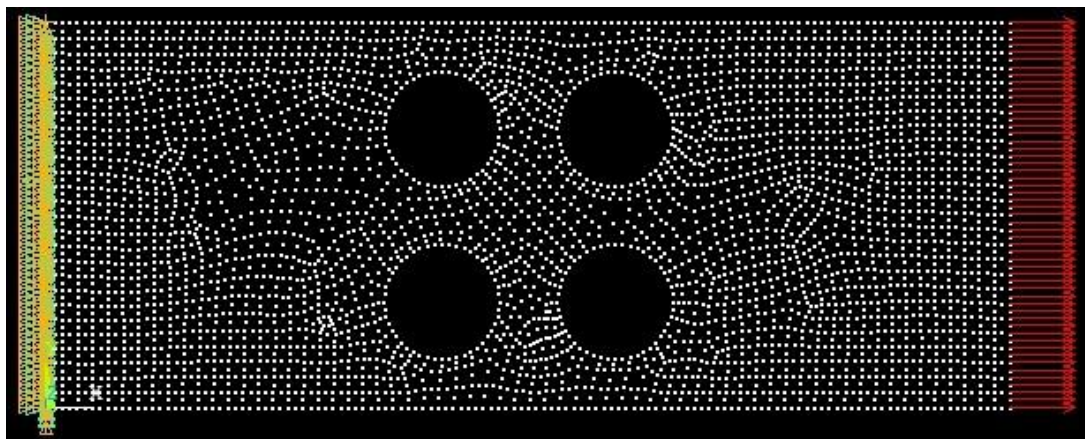


Fig.8: Square Pattern of Holes in Uniaxial Tensile Loading at one end only



Fig.9: Deflection for Plate with Square Pattern of Holes

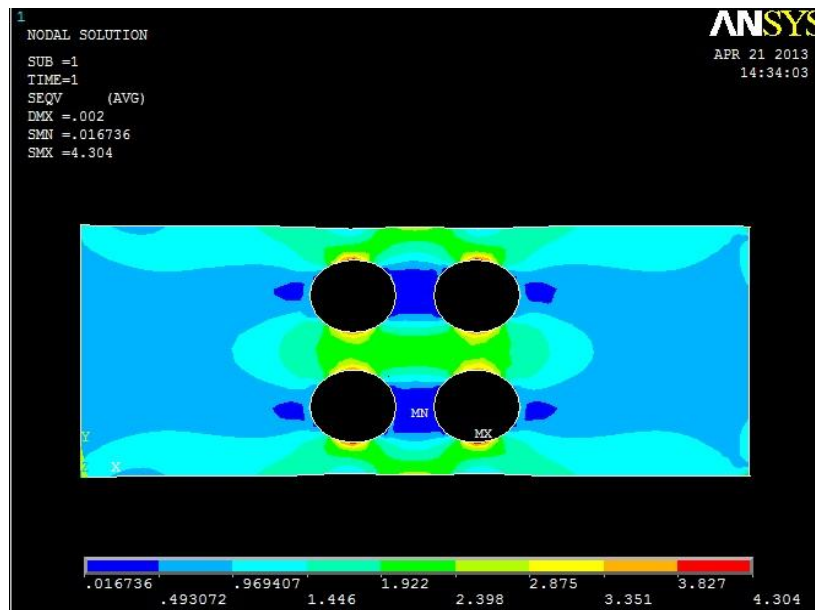


Fig.10: Von Mises Stresses for Plate with Square Pattern of Holes

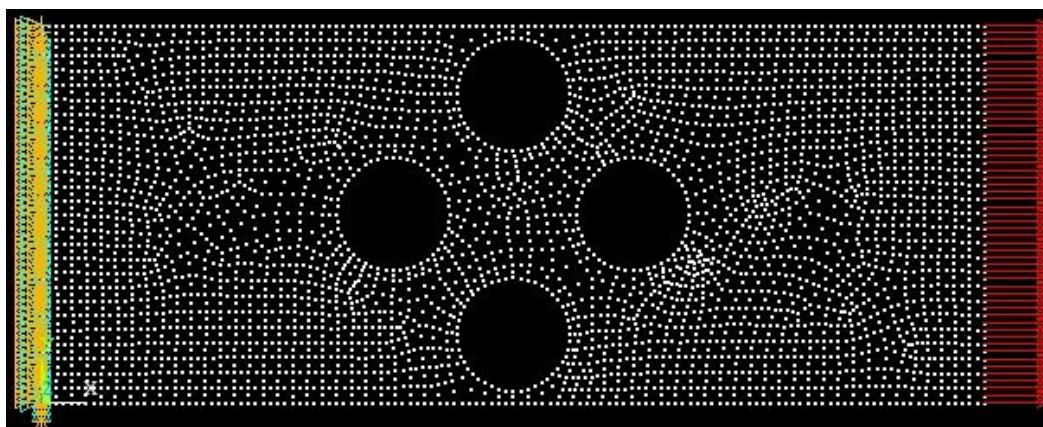


Fig.11: Diamond Pattern of Holes in Uniaxial Tensile Loading at one end only

V. Conclusion:

1. The SCF, Stress induced and the Deflections are not affected by the Tensile or Compressive loading when the loading directions and the boundary conditions are remains same for the same test specimen.
2. The deflection is maximum for uni-axial tensile or compressive loading at one end only while it is minimum for bi-axial tensile or compressive loading at both ends except for dia-30.48mm it is minimum for uni-axial tensile or compressive loading at both ends.
3. The stress obtained is maximum for bi-axial mixed loading while it is minimum for uni-axial tensile or compressive at both ends except for dia-76.20mm it is maximum for uni-axial tensile or compressive at one end only and minimum for bi-axial tensile or compressive at both ends.
4. The Analytical values of SCF and that obtained by FEA decreases exponentially with increase in d/w ratio but varies from each other by small negotiable amount in all conditions and the behavior of the graph remains same in all cases except in mixed loading which is almost linear.
5. In case study of patterns the stress induced in diamond pattern is almost two and half times that of induced in square pattern. The neck region is formed in diamond pattern at width end and length center where the possibility of failure is more. As there is no such neck region is formed the design of square pattern is preferred over that of design of diamond pattern.

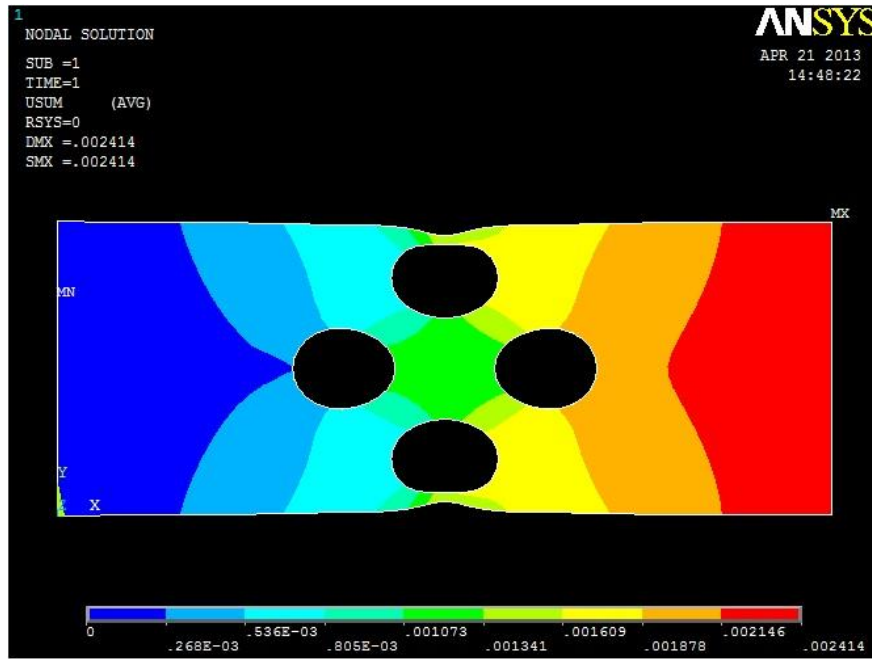


Fig.12: Deflection for Plate with Diamond Pattern of Holes

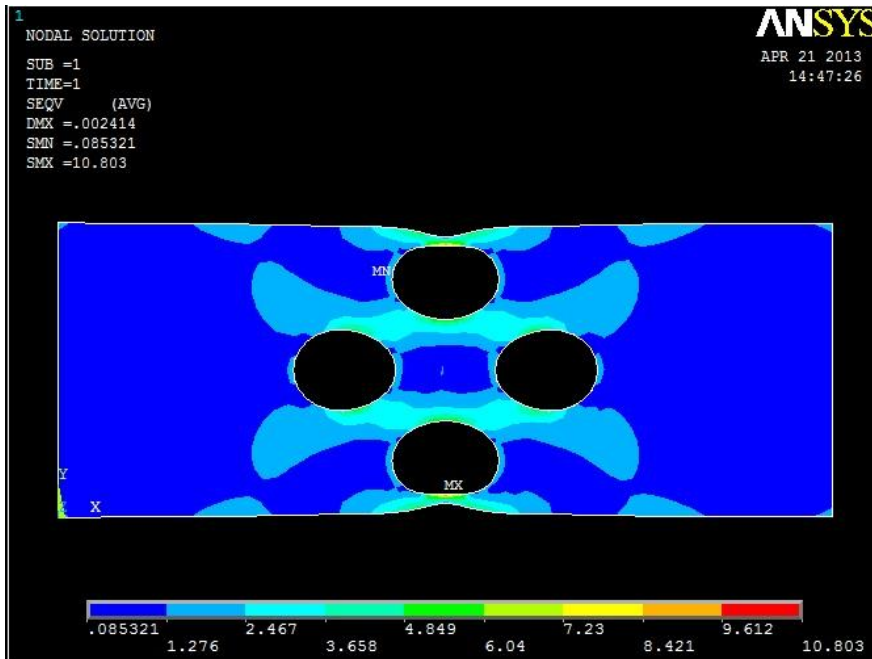


Fig.13: Von Misses Stresses for Plate with Diamond Pattern of Holes

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